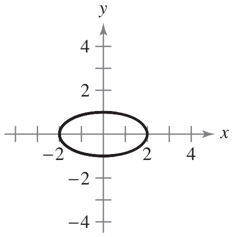
Review Exercises

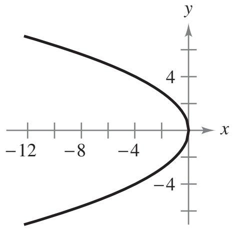
See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Matching **In Exercises 1–6, match the equation with the correct graph. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]**

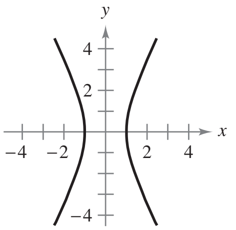
(a)



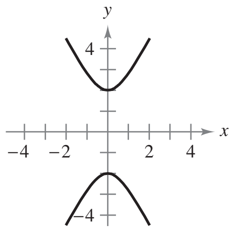
(b)



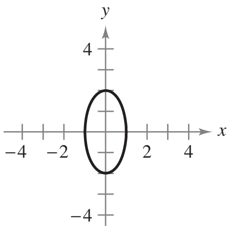
(c)



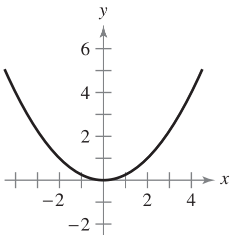
(d)



(e)



(f)



1. $4x^2 + y^2 = 4$

2. $4x^2 - y^2 = 4$

3. $y^2 = -4x$

4. $y^2 - 4x^2 = 4$

5. $x^2 + 4y^2 = 4$

6. $x^2 = 4y$

Identifying a Conic **In Exercises 7–14, identify the conic, analyze the equation (center, radius, vertices, foci, eccentricity, directrix, and asymptotes, if possible), and sketch its graph. Use a graphing utility to confirm your results.**

7. $16x^2 + 16y^2 - 16x + 24y - 3 = 0$

8. $y^2 - 12y - 8x + 20 = 0$

9. $3x^2 - 2y^2 + 24x + 12y + 24 = 0$

10. $5x^2 + y^2 - 20x + 19 = 0$

11. $3x^2 + 2y^2 - 12x + 12y + 29 = 0$

12. $12x^2 - 12y^2 - 12x + 24y - 45 = 0$

13. $x^2 - 6x - 8y + 1 = 0$

14. $9x^2 + 25y^2 + 18x - 100y - 116 = 0$

Finding an Equation of a Parabola **In Exercises 15 and 16, find an equation of the parabola.**

15. Vertex: $(0, 2)$
Directrix: $x = -3$

16. Vertex: $(2, 6)$
Focus: $(2, 4)$

Finding an Equation of an Ellipse **In Exercises 17–20, find an equation of the ellipse.**

17. Center: $(0, 0)$
Focus: $(5, 0)$
Vertex: $(7, 0)$

18. Center: $(0, 0)$
Major axis: vertical
Points on the ellipse: $(1, 2), (2, 0)$

19. Vertices: $(3, 1), (3, 7)$
Eccentricity: $\frac{2}{3}$

20. Foci: $(0, \pm 7)$
Major axis length: 20

Finding an Equation of a Hyperbola **In Exercises 21–24, find an equation of the hyperbola.**

21. Vertices: $(0, \pm 8)$
Asymptotes: $y = \pm 2x$

22. Vertices: $(\pm 2, 0)$
Asymptotes: $y = \pm 32x$

23. Vertices: $(\pm 7, -1)$
Foci: $(\pm 9, -1)$

24. Center: $(0, 0)$
Vertex: $(0, 3)$
Focus: $(0, 6)$

25. Satellite Antenna A cross section of a large parabolic antenna is modeled by the graph of

$$y = \frac{x^2}{200}, \quad -100 \le x \le 100.$$

The receiving and transmitting equipment is positioned at the focus.

(a) Find the coordinates of the focus.

(b) Find the surface area of the antenna.

26. Using an Ellipse Consider the ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1.$

(a) Find the area of the region bounded by the ellipse.

(b) Find the volume of the solid generated by revolving the region about its major axis.

Using Parametric Equations **In Exercises 27–34, sketch the curve represented by the parametric equations (indicate the orientation of the curve), and write the corresponding rectangular equation by eliminating the parameter.**

27. $x = 1 + 8t, \ y = 3 - 4t$

28. $x = t - 6, \ y = t^2$

29. $x = e^t - 1, \ y = e^{3t}$

30. $x = e^{4t}, \ y = t + 4$

31. $x = 6 \cos \theta, \ y = 6 \sin \theta$

32. $x = 2 + 5 \cos t, \ y = 3 + 2 \sin t$

33. $x = 2 + \sec \theta, \ y = 3 + \tan \theta$

34. $x = 5 \sin^3 \theta, \ y = 5 \cos^3 \theta$

Finding Parametric Equations **In Exercises 35 and 36, find two different sets of parametric equations for the rectangular equation.**

35. $y = 4x + 3$

36. $y = x^2 - 2$

37. Rotary Engine The rotary engine was developed by Felix Wankel in the 1950s. It features a rotor that is a modified equilateral triangle. The rotor moves in a chamber that, in two dimensions, is an epitrochoid. Use a graphing utility to graph the chamber modeled by the parametric equations

$x = \cos 3\theta + 5 \cos \theta$

and

$y = \sin 3\theta + 5 \sin \theta.$

38. Serpentine Curve Consider the parametric equations $x = 2 \cot \theta$ and $y = 4 \sin \theta \cos \theta$, $0 < \theta < \pi$.

(a) Use a graphing utility to graph the curve.

(b) Eliminate the parameter to show that the rectangular equation of the serpentine curve is $(4 + x^2)y = 8x$.

Finding Slope and Concavity **In Exercises 39–46, find dy/dx and d^2y/dx^2, and find the slope and concavity (if possible) at the given value of the parameter.**

	Parametric Equations	Parameter
39.	$x = 2 + 5t, \quad y = 1 - 4t$	$t = 3$
40.	$x = t - 6, \quad y = t^2$	$t = 5$
41.	$x = \dfrac{1}{t}, \quad y = 2t + 3$	$t = -1$
42.	$x = \dfrac{1}{t}, \quad y = t^2$	$t = -2$
43.	$x = 5 + \cos \theta, \quad y = 3 + 4 \sin \theta$	$\theta = \dfrac{\pi}{6}$
44.	$x = 10 \cos \theta, \quad y = 10 \sin \theta$	$\theta = \dfrac{\pi}{4}$
45.	$x = \cos^3 \theta, \quad y = 4 \sin^3 \theta$	$\theta = \dfrac{\pi}{3}$
46.	$x = e^t, \quad y = e^{-t}$	$t = 1$

Finding an Equation of a Tangent Line **In Exercises 47 and 48, (a) use a graphing utility to graph the curve represented by the parametric equations, (b) use a graphing utility to find $dx/d\theta$, $dy/d\theta$, and dy/dx at the given value of the parameter, (c) find an equation of the tangent line to the curve at the given value of the parameter, and (d) use a graphing utility to graph the curve and the tangent line from part (c).**

	Parametric Equations	Parameter
47.	$x = \cot \theta, \quad y = \sin 2\theta$	$\theta = \dfrac{\pi}{6}$
48.	$x = \dfrac{1}{4} \tan \theta, \quad y = 6 \sin \theta$	$\theta = \dfrac{\pi}{3}$

Horizontal and Vertical Tangency **In Exercises 49–52, find all points (if any) of horizontal and vertical tangency to the curve. Use a graphing utility to confirm your results.**

49. $x = 5 - t, \quad y = 2t^2$

50. $x = t + 2, \quad y = t^3 - 2t$

51. $x = 2 + 2 \sin \theta, \quad y = 1 + \cos \theta$

52. $x = 2 - 2 \cos \theta, \quad y = 2 \sin 2\theta$

Arc Length **In Exercises 53 and 54, find the arc length of the curve on the given interval.**

	Parametric Equations	Interval
53.	$x = t^2 + 1, \quad y = 4t^3 + 3$	$0 \le t \le 2$
54.	$x = 6 \cos \theta, \quad y = 6 \sin \theta$	$0 \le \theta \le \pi$

Surface Area **In Exercises 55 and 56, find the area of the surface generated by revolving the curve about (a) the x-axis and (b) the y-axis.**

55. $x = t, \quad y = 3t, \quad 0 \le t \le 2$

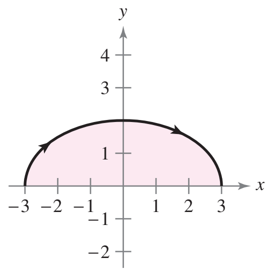
56. $x = 2 \cos \theta, \quad y = 2 \sin \theta, \quad 0 \le \theta \le \dfrac{\pi}{2}$

Area **In Exercises 57 and 58, find the area of the region.**

57. $x = 3 \sin \theta$

$y = 2 \cos \theta$

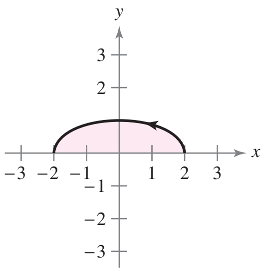
$-\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2}$



58. $x = 2 \cos \theta$

$y = \sin \theta$

$0 \le \theta \le \pi$



Polar-to-Rectangular Conversion **In Exercises 59–62, plot the point in polar coordinates and find the corresponding rectangular coordinates of the point.**

59. $\left(5, \dfrac{3\pi}{2}\right)$

60. $\left(-6, \dfrac{7\pi}{6}\right)$

61. $\left(\sqrt{3}, 1.56\right)$

62. $(-2, -2.45)$

Rectangular-to-Polar Conversion **In Exercises 63–66, the rectangular coordinates of a point are given. Plot the point and find *two* sets of polar coordinates of the point for $0 \le \theta < 2\pi$.**

63. $(4, -4)$

64. $(0, -7)$

65. $(-1, 3)$

66. $\left(-\sqrt{3}, -\sqrt{3}\right)$

Rectangular-to-Polar Conversion **In Exercises 67–72, convert the rectangular equation to polar form and sketch its graph.**

67. $x^2 + y^2 = 25$

68. $x^2 - y^2 = 4$

69. $y = 9$

70. $x = 6$

71. $x^2 = 4y$

72. $x^2 + y^2 - 4x = 0$

Polar-to-Rectangular Conversion In Exercises 73–78, convert the polar equation to rectangular form and sketch its graph.

73. $r = 3\cos\theta$
74. $r = 10$
75. $r = 6\sin\theta$
76. $r = 3\csc\theta$
77. $r = -2\sec\theta\tan\theta$
78. $\theta = \dfrac{3\pi}{4}$

Graphing a Polar Equation In Exercises 79–82, use a graphing utility to graph the polar equation.

79. $r = \dfrac{3}{\cos(\theta - \pi/4)}$
80. $r = 2\sin\theta\cos^2\theta$
81. $r = 4\cos 2\theta\sec\theta$
82. $r = 4(\sec\theta - \cos\theta)$

Horizontal and Vertical Tangency In Exercises 83 and 84, find the points of horizontal and vertical tangency (if any) to the polar curve.

83. $r = 1 - \cos\theta$
84. $r = 3\tan\theta$

Tangent Lines at the Pole In Exercises 85 and 86, sketch a graph of the polar equation and find the tangents at the pole.

85. $r = 4\sin 3\theta$
86. $r = 3\cos 4\theta$

Sketching a Polar Graph In Exercises 87–96, sketch a graph of the polar equation.

87. $r = 6$
88. $\theta = \dfrac{\pi}{10}$
89. $r = -\sec\theta$
90. $r = 5\csc\theta$
91. $r^2 = 4\sin^2 2\theta$
92. $r = 3 - 4\cos\theta$
93. $r = 4 - 3\cos\theta$
94. $r = 4\theta$
95. $r = -3\cos 2\theta$
96. $r = \cos 5\theta$

Finding the Area of a Polar Region In Exercises 97–102, find the area of the region.

97. One petal of $r = 3\cos 5\theta$
98. One petal of $r = 2\sin 6\theta$
99. Interior of $r = 2 + \cos\theta$
100. Interior of $r = 5(1 - \sin\theta)$
101. Interior of $r^2 = 4\sin 2\theta$
102. Common interior of $r = 4\cos\theta$ and $r = 2$

Finding the Area of a Polar Region In Exercises 103–106, use a graphing utility to graph the polar equation. Find the area of the given region analytically.

103. Inner loop of $r = 3 - 6\cos\theta$
104. Inner loop of $r = 2 + 4\sin\theta$
105. Between the loops of $r = 3 - 6\cos\theta$
106. Between the loops of $r = 2 + 4\sin\theta$

Finding Points of Intersection In Exercises 107 and 108, find the points of intersection of the graphs of the equations.

107. $r = 1 - \cos\theta$
$r = 1 + \sin\theta$

108. $r = 1 + \sin\theta$
$r = 3\sin\theta$

Finding the Arc Length of a Polar Curve In Exercises 109 and 110, find the length of the curve over the given interval.

	Polar Equation	Interval
109.	$r = 5\cos\theta$	$\dfrac{\pi}{2} \le \theta \le \pi$
110.	$r = 3(1 - \cos\theta)$	$0 \le \theta \le \pi$

Finding the Area of a Surface of Revolution In Exercises 111 and 112, write an integral that represents the area of the surface formed by revolving the curve about the given line. Use the integration capabilities of a graphing utility to approximate the integral accurate to two decimal places.

	Polar Equation	Interval	Axis of Revolution
111.	$r = 1 + 4\cos\theta$	$0 \le \theta \le \dfrac{\pi}{2}$	Polar axis
112.	$r = 2\sin\theta$	$0 \le \theta \le \dfrac{\pi}{2}$	$\theta = \dfrac{\pi}{2}$

Sketching and Identifying a Conic In Exercises 113–118, find the eccentricity and the distance from the pole to the directrix of the conic. Then sketch and identify the graph. Use a graphing utility to confirm your results.

113. $r = \dfrac{6}{1 - \sin\theta}$
114. $r = \dfrac{2}{1 + \cos\theta}$
115. $r = \dfrac{6}{3 + 2\cos\theta}$
116. $r = \dfrac{4}{5 - 3\sin\theta}$
117. $r = \dfrac{4}{2 - 3\sin\theta}$
118. $r = \dfrac{8}{2 - 5\cos\theta}$

Finding a Polar Equation In Exercises 119–124, find a polar equation for the conic with its focus at the pole. (For convenience, the equation for the directrix is given in rectangular form.)

	Conic	Eccentricity	Directrix
119.	Parabola	$e = 1$	$x = 4$
120.	Ellipse	$e = \dfrac{3}{4}$	$y = -2$
121.	Hyperbola	$e = 3$	$y = 3$

	Conic	Vertex or Vertices
122.	Parabola	$\left(2, \dfrac{\pi}{2}\right)$
123.	Ellipse	$(5, 0), (1, \pi)$
124.	Hyperbola	$(1, 0), (7, 0)$